A system with reversible equilibria

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We track the motion of a small ball rolling within a machined confining groove as the apparatus is shaken and tilted harmonically, using a high-speed digital camera. The tilting effectively reverses the stability of the equilibria in a periodic fashion. The equations of motion can be written as:

\[
\left[ (X'(s))^2 + Y'(s)^2 \frac{d^2 s}{dt^2} - X'(s) \frac{d^2 f}{dt^2} \right] + \left( X'(s)X''(s) + Y'(s)Y''(s) \right) (\frac{ds}{dt})^2 + \beta (X'(s)^2 + Y'(s)^2) \frac{ds}{dt} - Y(s)Y'(s) (\frac{d\phi}{dt})^2 + gY'(s) \cos \phi = 0
\]

This system is most conveniently analyzed using an arc-length coordinate \( s \), damping \( \beta \), gravity \( g \), and horizontal forcing \( f(t) \) and tilt angle \( \phi(t) \).
The frequency content of typical responses changes as a function of the forcing period as shown in the spectrograms below.

Some preliminary numerical results:

Some preliminary experimental results:

Poincaré sections taking at a prescribed forcing phase:

Main issue: comparing and contrasting results for (i) non-tilting, (ii) tilting about a non-zero mean, and (iii) tilting about a zero mean. Manipulating the equilibrium reversal can be used (perhaps) to influence the extent of a basin of attraction.