A system with reversible equilibria

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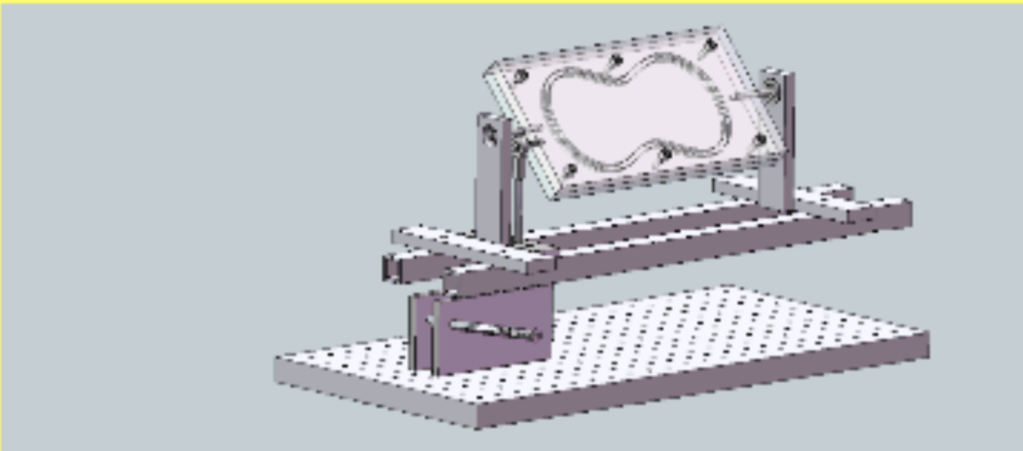
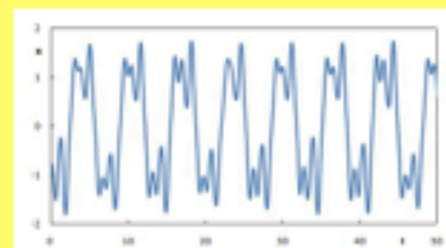


We track the motion of a small ball rolling within a machined confining groove as the apparatus is shaken and tilted harmonically, using a high-speed digital camera. The tilting effectively reverses the stability of the equilibria in a periodic fashion. The equations of motion can be written as:

$$\left[(X'(s)^2 + Y'(s)^2)\frac{d^2 s}{dt^2} - X'(s)\frac{d^2 f}{dt^2}\right] + \left(X'(s)X''(s) + Y'(s)Y''(s)\right)\left(\frac{ds}{dt}\right)^2 + \beta(X'(s)^2 + Y'(s)^2)\frac{ds}{dt} - Y(s)Y'(s)\left(\frac{d\phi}{dt}\right)^2 + gY'(s)\cos\phi = 0$$

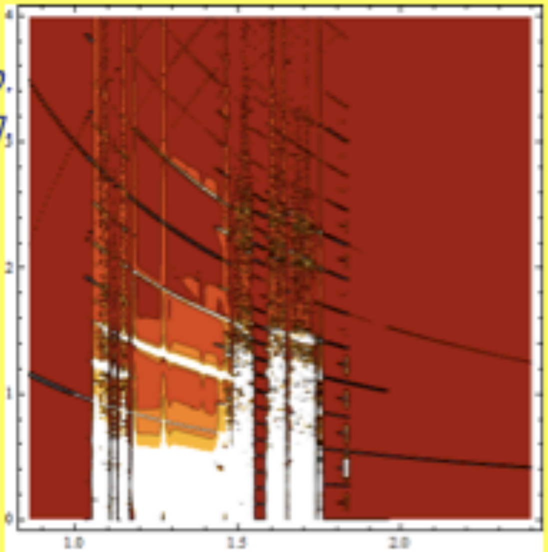


(x,t)

This system is most conveniently analyzed using an arc-length coordinate *s*, damping β, gravity *g*, and horizontal forcing *f(t)* and tilt angle $\phi(t)$.

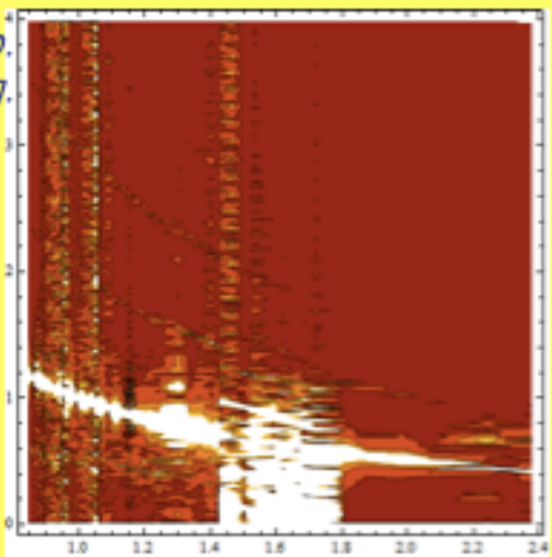
The frequency content of typical responses changes as a function of the forcing period as shown in the spectrograms below.

resp. freq.

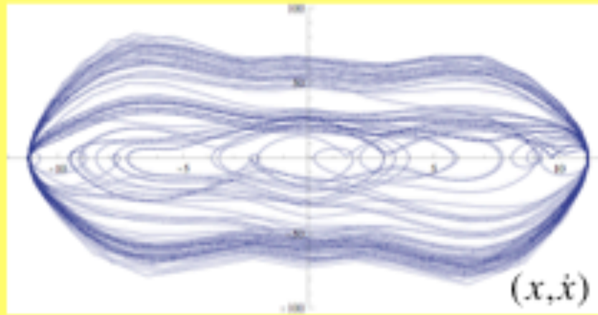


forcing period

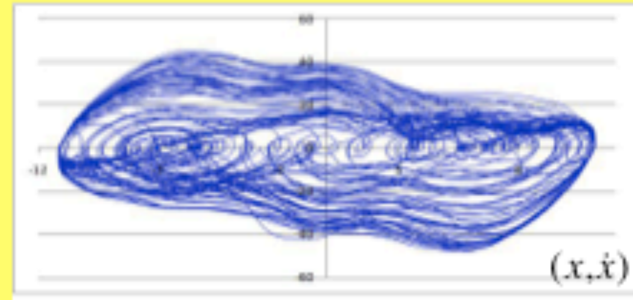
resp. freq.



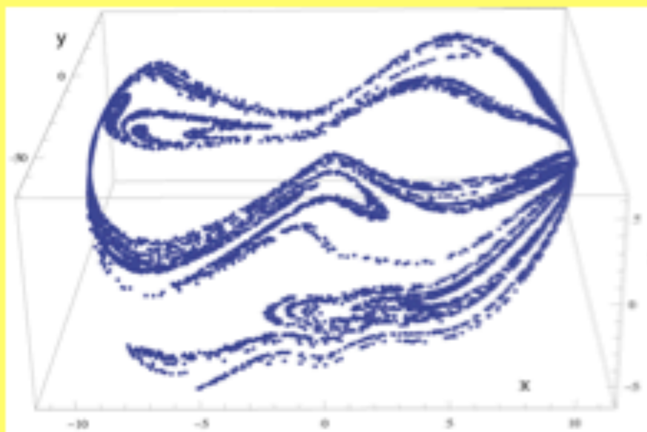
Some preliminary numerical results:

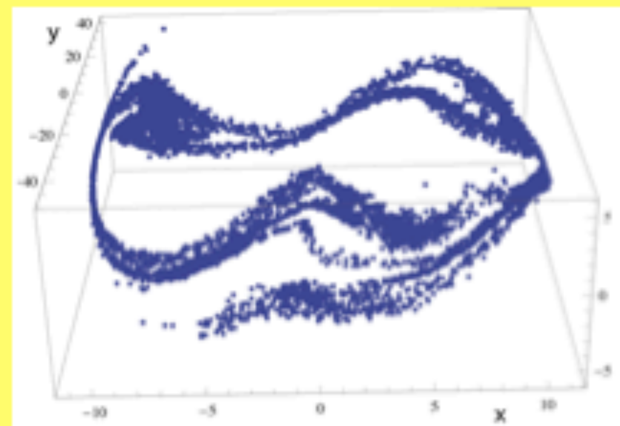


Some preliminary experimental results:



Poincaré sections taking at a prescribed forcing phase:





Main issue: comparing and contrasting results for (i) non-tilting, (ii) tilting about a non-zero mean, and (iii) tilting about a zero mean. Manipulating the equilibrium reversal can be used (perhaps) to influence the extent of a basin of attraction.